Traffic Flow Optimization at Bundaran Simpang Bandara Batam Using Gauss-Jordan Elimination

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***Abstract*— Batam, the largest city in the Riau Archipelago, plays a significant role in Indonesia's economic development. Simpang Bandara roundabout, a critical intersection in Batam, is essential for managing traffic flow across the city's growing infrastructure. This paper analyzes the traffic flow at the roundabout using systems of linear equations and Gauss-Jordan elimination. Two scenarios are examined: one with flexibility in traffic flow through parametric solutions and another with congestion due to over-constrained systems. The findings highlight the significant impact of vehicle volumes on network efficiency and underscore the need for dynamic traffic management and infrastructure planning to optimize flow at key intersections such as roundabouts.**

***Keywords*— Gauss-Jordan Elimination, Optimize, Systems of Linear Equations, Traffic Flow.**

I. INTRODUCTION

Traffic flow optimization is a critical aspect of urban planning and transportation engineering, especially in rapidly growing cities like Batam. Bundaran Simpang Bandara Batam, a vital node in the city's transportation network, plays a key role in managing traffic for people departing to or arriving from Hang Nadim International Airport. As a major point of convergence, it connects key areas while handling significant traffic volumes daily.

This paper proposes a solution using Gauss-Jordan Elimination to analyze and optimize traffic flow at Bundaran Simpang Bandara Batam. By applying this mathematical method to solve systems of linear equations representing traffic flow, the relationships between incoming and outgoing vehicles at the roundabout can be determined. This approach enables the identification of flexible flow configurations, as well as potential inefficiencies or congestion points in the current traffic system.

The concept that is used in this paper are not only applicable to Bundaran Simpang Bandara Batam but can also be utilized in various traffic systems worldwide. It is hoped that this study can serve as a guide for urban planners and engineers to design or improve road networks, ensuring smoother traffic flow and reducing congestion in critical areas.

II. THEORETICAL BASIS

A. Matrix

Matrix is a rectangular array or table (also called array of array) made up of numbers, symbols, points, or characters as its elements, arranged into rows and columns. The dimensions of matrix are represented as the number of rows multiplied by the number of columns (rows × columns). A matrix is typically written as $P_{m\times n}$, where P is the matrix, m is the number of rows, and n is the number of columns.

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Fig 2.1 Matrix.
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-01-Review-Matriks-2023.pdf

Matrix is widely used in mathematics, particularly in solving systems of linear equations, among many other applications. Matrix can also be manipulated through mathematical operations like addition, subtraction, scalar multiplication, and multiplication. These operations combine elements from two matrices to produce a new matrix, with each element reflecting the result of the operation.

B. System of Linear Equations

System of linear equations is a group of two or more equations that use the same set of variables. These equations show how variables are related to each other and often represent lines, planes, or other shapes. The solution to the system is the set of values for the variables that works for all the equations at the same time. This solution usually represents where the lines or shapes intersect or overlap.

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \\ \vdots \quad\quad \vdots \quad\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Fig 2.2 System of linear equations in the form of matrix.
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-01-Review-Matriks-2023.pdf

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Fig 2.3 System of linear equations in the form of matrix multiplication.
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-01-Review-Matriks-2023.pdf

System of linear equations can have three possible outcomes: one solution (unique), infinitely many solutions, or no solution. If there is one solution, it means a single point satisfies all the equations in the system. If there are infinitely many solutions, it means an endless number of points meet the conditions of the system. If there is no solution, it means that no points satisfy all the equations simultaneously.

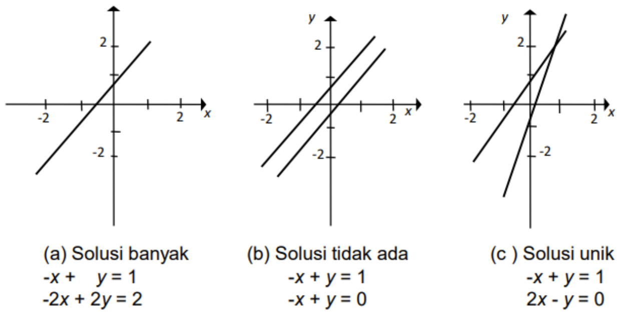


Fig 2.4 System of linear equations in two variables with (a) many solutions, (b) no solution, (c) one solution.
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-04-Tiga-Kemungkinan-Solusi-SPL-2023.pdf

System of linear equations can also be represented as an augmented matrix.

$$\begin{aligned} x_1 + 3x_2 - 6x_3 &= 9 \\ 2x_1 - 6x_2 + 4x_3 &= 7 \\ 5x_1 + 2x_2 - 5x_3 &= -2 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 1 & 3 & -6 & 9 \\ 2 & -6 & 4 & 7 \\ 5 & 2 & -5 & -2 \end{bmatrix}$$

Fig 2.5 System of linear equations as an augmented matrix
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-03-Sistem-Persamaan-Linier-2023.pdf

There are three elementary row operations that can be performed to an augmented matrix. These operations will not change the value of the matrix.

1. Interchange the positions of any two rows.

$$\begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 3 \\ 7 & 1 & 2 \end{bmatrix} \leftrightarrow \begin{bmatrix} 4 & 8 & 3 \\ 2 & 4 & 5 \\ 7 & 1 & 2 \end{bmatrix}$$

Fig 2.6 Interchange the 1st row and the 2nd row
Source: https://www.geeksforgeeks.org/elementary-operations-on-matrices/

2. Multiply any row by a non-zero constant.

$$\begin{bmatrix} 2 & 4 & 5 \\ 4 \times 2 & 8 \times 2 & 3 \times 2 \\ 7 & 1 & 2 \end{bmatrix} \leftrightarrow \begin{bmatrix} 2 & 4 & 5 \\ 8 & 16 & 6 \\ 7 & 1 & 2 \end{bmatrix}$$

Fig 2.7 Multiply the 2nd row by two
Source: https://www.geeksforgeeks.org/elementary-operations-on-matrices/

3. Add a multiple (can also be one) of one row to another row.

$$\begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 3 \\ 7+2 & 1+4 & 2+5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 3 \\ 9 & 5 & 7 \end{bmatrix}$$

Fig 2.8 Adding the 3rd row with the 1st row
Source: https://www.geeksforgeeks.org/elementary-operations-on-matrices/

C. Gaussian Elimination

Gaussian elimination is a method of solving systems of linear equations by manipulating matrix into row echelon form with the help of elementary row operations. A matrix is in row echelon form if it satisfies these following conditions:

1. For each row, the first non-zero number is 1 (called a leading 1 or pivot).
2. Any rows consisting entirely of zeros will be placed at the bottom of the matrix.
3. Each pivot in the higher row is further left than the pivot in the lower row.

$$\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & * & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & * & * & * \\ 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Keterangan: * adalah sembarang nilai

Fig 2.9 Row echelon form
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-02-Matriks-Eselon-2023.pdf

There are three steps to solve systems of linear equations using Gaussian elimination:

1. Represent systems of linear equations as an augmented matrix.
2. Perform elementary row operations on the augmented matrix until it reaches row echelon form.
3. Apply backward substitution to solve the equation.

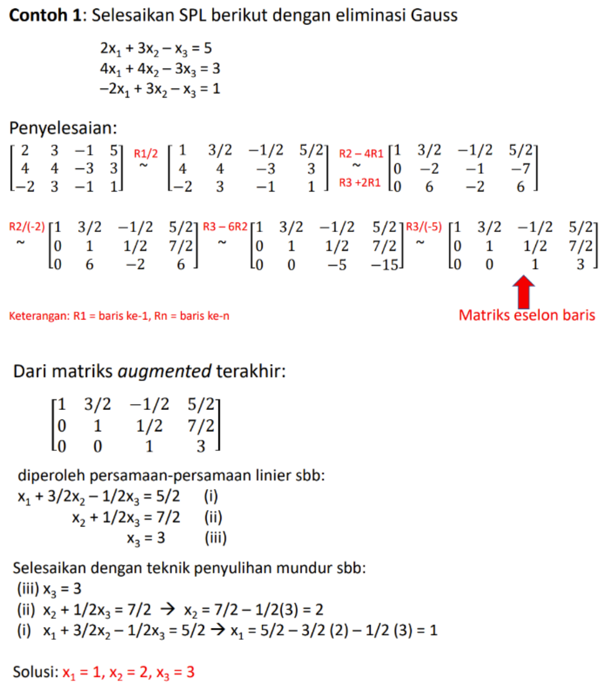


Fig 2.10 Solving systems of linear equations using Gaussian elimination
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-03-Sistem-Persamaan-Linier-2023.pdf

D. Gauss-Jordan Elimination

Gauss-Jordan elimination is developed from Gaussian elimination method. The main difference in Gauss-Jordan elimination is manipulating the matrix into reduced row-echelon form, which means not only the elements below the pivot become zero, but the elements above the pivot are also zero.

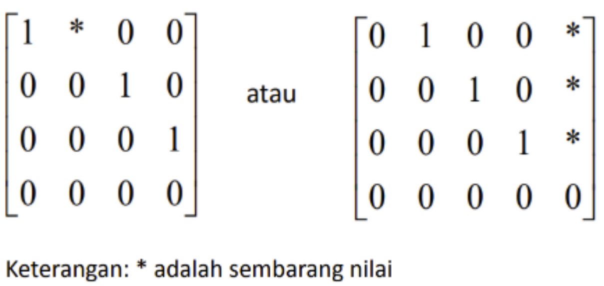


Fig 2.11 Reduced row-echelon form
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-02-Matriks-Eselon-2023.pdf

The steps for solving systems of linear equations using Gauss-Jordan elimination are similar to Gaussian elimination. However, the Gauss-Jordan elimination method involves two phases:

1. Forward phase, which eliminates the values below the pivot by producing zeros.
2. Backward phase, which eliminates the values above the pivot by producing zeros.

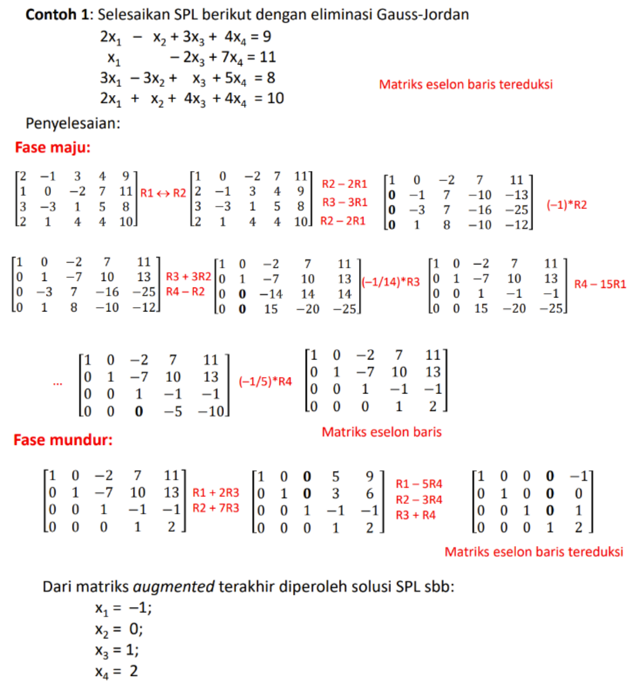


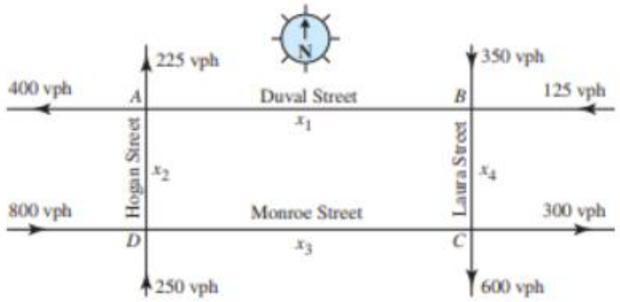
Fig 2.12 Solving systems of linear equations using Gauss-Jordan elimination
Source: https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-05-Sistem-Persamaan-Linier-2-2023.pdf

E. Traffic Flow

Traffic flow refers to the movement of vehicles through transportation routes and infrastructure, such as roads, highways, roundabouts, control devices, and signage. It is essentially a way to measure how many vehicles pass a certain point over a given period. The goal of traffic flow is to understand and improve transportation networks by making traffic flow more efficient and reducing congestion. There are several factors that play a role in traffic flow:

1. Flow rate (volume), refers to the number of vehicles passing through a specific point of a road.
2. Speed, refers to how fast vehicles are traveling.
3. Density, refers to the concentration of vehicles at a specific point of a road.
4. Capacity, refers to the maximum number of vehicles that a road can effectively handle.

Traffic flow can also be modeled as a system of linear equations.



Intersection A: Traffic in = $x_1 + x_2$.
Traffic out = 400 + 225. Thus $x_1 + x_2 = 625$.

Intersection B: Traffic in = 350 + 125.
Traffic out = $x_1 + x_4$. Thus $x_1 + x_4 = 475$.

Intersection C: Traffic in = $x_3 + x_4$.
Traffic out = 600 + 300. Thus $x_3 + x_4 = 900$.

Intersection D: Traffic in = 800 + 250.
Traffic out = $x_2 + x_3$. Thus $x_2 + x_3 = 1050$.

These constraints on the traffic are described by the following system of linear equations:

$$\begin{aligned} x_1 + x_2 &= 625 \\ x_1 + x_4 &= 475 \\ x_3 + x_4 &= 900 \\ x_2 + x_3 &= 1050 \end{aligned}$$

Fig 2.13 Traffic flow modelling as a system of linear equations
Source: https://math.jbpub.com/advancedengineering/docs/Project8.2_TrafficFlow.pdf

The system of linear equations can be analyzed by solving the equations. One of the approaches to solve these equations is by using Gaussian elimination or Gauss-Jordan elimination. If the system has a unique solution, it shows that the road network operates with a stable flow, ensuring consistent traffic at all intersections. If the system has multiple solutions, it shows that the road network supports flexible flow configurations with multiple possible traffic patterns. However, if the system has no solution, it shows that the road is poorly designed and may experience congestion and other traffic issues.

III. Implementation

A. Research Limitation

On writing this paper, the author uses some limitations to optimize the traffic flow at Bundaran Simpang Bandara Batam using Gauss-Jordan elimination. The limitations are:

1. The number of vehicles entering and exiting are based on assumptions.
2. The total number of vehicles entering the roundabout equals the total number of vehicles exiting.
3. Traffic flow within the roundabout is constant.
4. All traffic flow values are guaranteed to be positive.

B. Research Site

The location of research that is used in this paper is Bundaran Simpang Bandara, located in Batam Island, Riau Archipelago, Indonesia.

Fig 3.1 Bundaran Simpang Bandara Batam
Source: Google Maps

C. Traffic Flow Analysis at Bundaran Simpang Bandara Batam Using Gauss-Jordan Elimination

To effectively model the traffic flow at Bundaran Simpang Bandara Batam, as shown in *Fig 3.1*, the roads that connect to the roundabout need to be identified and classified as entry points and exit points. Each intersection can be represented as a node and the flow of traffic (in vehicles per unit time) can be modeled by variables or fixed values.

Let's assume there are 6 variables: x_1, x_2, x_3, x_4, x_5, x_6, and 6 fixed values: *60, 80, 100, 120, 140, 160*. These variables and values are plotted at Bundaran Simpang Bandara Batam, as shown in *Fig 3.2*.

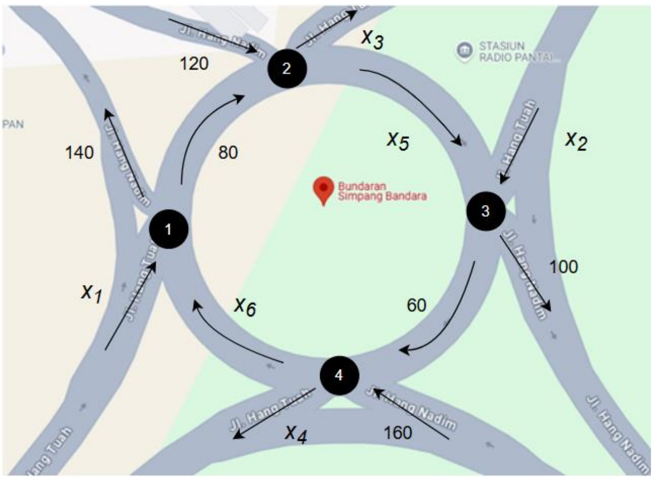


Fig 3.2 Traffic Flow at Bundaran Simpang Bandara Batam

There are four intersections that are marked with black color nodes, as shown in *Fig 3.2*. For the first intersection, the input are x_1 and x_6, and the output are *80* and *140*. For the second intersection, the input are *80* and *120*, and the output are x_3 and x_5. For the third intersection, the input are x_2 and x_5, and the output are *60* and *100*. For the fourth intersection, the input are *60* and *160*, and the output are x_4 and x_6.

All four intersections can be represented as the following system of linear equations:

$$x_1 + x_6 = 80 + 140 \dots (1)$$
$$80 + 120 = x_3 + x_5 \dots (2)$$
$$x_2 + x_5 = 60 + 100 \dots (3)$$
$$60 + 160 = x_4 + x_6 \dots (4)$$

which simplifies to:

$$x_1 + x_6 = 220 \dots (1)$$
$$x_3 + x_5 = 200 \dots (2)$$
$$x_2 + x_5 = 160 \dots (3)$$
$$x_4 + x_6 = 220 \dots (4)$$

After formulating the system of linear equations, convert it as an augmented matrix and apply Gauss-Jordan elimination to solve the problem.

Step 1: Write the system as an augmented matrix.

	x_1	x_2	x_3	x_4	x_5	x_6	b
1	1	0	0	0	0	1	220
2	0	0	1	0	1	0	200
3	0	1	0	0	1	0	160
4	0	0	0	1	0	1	220

Step 2: Find the pivot in the 1st column in the 1st row.

	x_1	x_2	x_3	x_4	x_5	x_6	b
1	1	0	0	0	0	1	220
2	0	0	1	0	1	0	200
3	0	1	0	0	1	0	160
4	0	0	0	1	0	1	220

Step 3: Find the pivot in the 2nd column and swap the 3rd and the 2nd row.

	x_1	x_2	x_3	x_4	x_5	x_6	b
1	1	0	0	0	0	1	220
2	0	1	0	0	1	0	160
3	0	0	1	0	1	0	200
4	0	0	0	1	0	1	220

Step 4: Find the pivot in the 3rd column in the 3rd row.

	x_1	x_2	x_3	x_4	x_5	x_6	b
1	1	0	0	0	0	1	220
2	0	1	0	0	1	0	160
3	0	0	1	0	1	0	200
4	0	0	0	1	0	1	220

Step 5: Find the pivot in the 4th column in the 4th row.

	x_1	x_2	x_3	x_4	x_5	x_6	b
1	1	0	0	0	0	1	220
2	0	1	0	0	1	0	160
3	0	0	0	0	1	0	200
4	0	0	0	1	0	1	220

For each row, the pivot has already been identified, so the Gauss-Jordan elimination process stops here. The solution to the problem can be represented as:

$$x_1 = 220 - x_6$$
$$x_2 = 160 - x_5$$
$$x_3 = 200 - x_5$$
$$x_4 = 220 - x_6$$
$$x_5 = x_5$$
$$x_6 = x_6$$

which shows that x_5 and x_6 are free variables.

Let's try another assumption. In this case, the exact number of vehicles entering and exiting are known. Let's assume there are 4 variables: x_1, x_2, x_3, x_4, and 8 fixed values: *70*, *90*, *100*, *120*, *140*, *160*, *180*, *200*. These variables and values are plotted at Bundaran Simpang Bandara Batam, as shown in *Fig 3.3*.

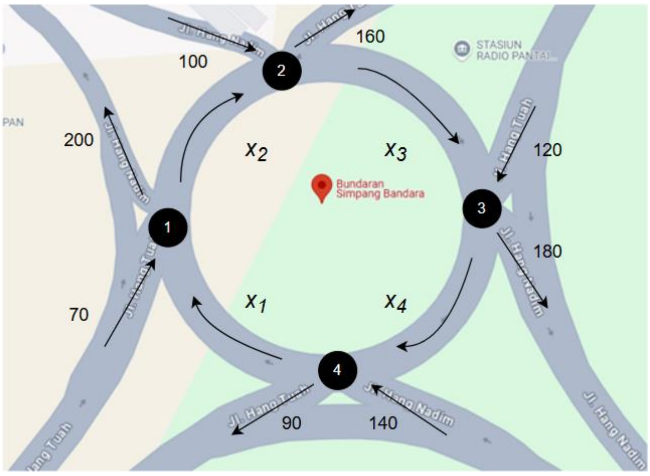


Fig 3.3 Traffic Flow at Bundaran Simpang Bandara Batam

There are four intersections that are marked with black color nodes, as shown in *Fig 3.3*. For the first intersection, the input are x_1 and *70*, and the output are x_2 and *200*. For the second intersection, the input are x_2 and *100*, and the output are x_3 and *160*. For the third intersection, the input are x_3 and *120*, and the output are x_4 and *180*. For the fourth intersection, the input are x_4 and *140*, and the output are x_1 and *90*.

All four intersections can be represented as the following system of linear equations:

$$x_1 + 70 = x_2 + 200 \dots (1)$$
$$x_2 + 100 = x_3 + 160 \dots (2)$$
$$x_3 + 120 = x_4 + 180 \dots (3)$$
$$x_4 + 140 = x_1 + 90 \dots (4)$$

which simplifies to:

$$x_1 – x_2 = 130 \dots (1)$$
$$x_2 – x_3 = 60 \dots (2)$$
$$x_3 – x_4 = 60 \dots (3)$$
$$x_4 – x_1 = -50 \dots (4)$$

After formulating the system of linear equations, convert it as an augmented matrix and apply Gauss-Jordan elimination to solve the problem.

Step 1: Write the system as an augmented matrix.

	x_1	x_2	x_3	x_4	b
1	1	-1	0	0	130
2	0	1	-1	0	60
3	0	0	1	-1	60
4	-1	0	0	1	-50

Step 2: Find the pivot in the 1st column in the 1st row.

	x_1	x_2	x_3	x_4	b
1	1	-1	0	0	130
2	0	1	-1	0	60
3	0	0	1	-1	60
4	-1	0	0	1	-50

Step 3: Eliminate the 1st column.

	x_1	x_2	x_3	x_4	b
1	1	-1	0	0	130
2	0	1	-1	0	60
3	0	0	1	-1	60
4	0	-1	0	1	80

Step 4: Find the pivot in the 2nd column in the 2nd row.

	x_1	x_2	x_3	x_4	b
1	1	-1	0	0	130
2	0	1	-1	0	60
3	0	0	1	-1	60
4	0	-1	0	1	80

Step 5: Eliminate the 2nd column.

	x_1	x_2	x_3	x_4	b
1	1	0	-1	0	190
2	0	1	-1	0	60
3	0	0	1	-1	60
4	0	0	-1	1	140

Step 6: Find the pivot in the 3rd column in the 3rd row.

	x_1	x_2	x_3	x_4	b
1	1	0	-1	0	190
2	0	1	-1	0	60
3	0	0	1	-1	60
4	0	0	-1	1	140

Step 7: Eliminate the 3rd column.

	x_1	x_2	x_3	x_4	b
1	1	0	0	-1	250
2	0	1	0	-1	120
3	0	0	1	-1	60
4	0	0	0	0	200

For each row, the pivot has already been identified, so the Gauss-Jordan elimination process stops here. As shown in the last process, a matrix row containing all zero coefficient entries and a non-zero constant entry is produced. This indicates that the system has no solutions.

IV. Analysis

Based on the calculation result using Gauss-Jordan Elimination, there are two cases. In the first case, there are infinitely many solutions leading to a parametric solution (where the variables ≥ 0). It happens because the system of equations is undetermined (has more variables than independent equations). The system doesn't have enough constraints to uniquely determine all the traffic flows. This reflects how traffic systems in real life can often adapt to different flow patterns. A roundabout might allow various distributions of vehicles across its roads, depending on traffic conditions at different times of the day. The parametric solution shows that the roundabout can accommodate multiple flow configurations. This flexibility can be an advantage in handling fluctuating traffic demands or unexpected changes, such as detours or temporary road closures. However, in the second case, there is no solution. It happens because the system of equations is over constrained (has more constraints than the system can satisfy). This leads to a no-solution scenario, indicating that the system is infeasible. Certain roads or lanes might be over-capacitated, leading to congestion or a mathematical contradiction in the equations. This inconsistency reflects a mismatch between the actual traffic conditions and the assumptions or design of the network. The no-solution scenario corresponds to real-world situations where a roundabout or traffic network is poorly designed. If a road leading into the roundabout has a higher inflow than the roundabout can handle, congestion builds up, and the system fails to function as intended. It shows that the roundabout cannot handle the specified traffic demands without significant delays or redesigning its infrastructure.

These two cases demonstrate how even slight changes in system constraints, traffic flow patterns, or the number of vehicles flowing through can significantly impact the overall functionality of a traffic network. In the first case, the system's flexibility allows for multiple configurations, making it adaptable to fluctuating traffic conditions. However, in the second case, an over-constrained system highlights how exceeding the network's capacity can lead to congestion and failure. The number of vehicles flowing through a roundabout plays a crucial role in determining whether the system can function smoothly or break down under pressure. High traffic volumes, if not managed properly, can overwhelm the network, causing delays, congestion, and inefficiencies. These scenarios emphasize the importance of dynamic traffic management strategies and infrastructure designed to handle varying traffic volumes while maintaining optimal flow.

V. Conclusion

In conclusion, Gauss-Jordan elimination offers a practical way to analyze traffic flow. This method helps identify the traffic situation and provides useful information. This information can be applied to improve traffic flow by adjusting signal timings, rerouting vehicles, or planning road expansions. Making upgrades to infrastructure and using real-time traffic monitoring systems can also boost efficiency and make roads safer. Additionally, these measures can contribute to reducing fuel consumption, lowering emissions, and improving overall urban mobility. However, it is essential to pair these methods with continuous monitoring and adaptability to address changing traffic patterns effectively. The insights gained here can also guide future urban planning efforts and ensure sustainable traffic solutions for growing cities.

VI. APPENDIX

Video explanation of this paper: https://youtu.be/9riF_SOM36w

VII. ACKNOWLEDGMENT

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 31 Desember 2024

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